

1. (a) Find the binomial expansion of

$$(4 + 5x)^{\frac{1}{2}}, \quad |x| < \frac{4}{5}$$

in ascending powers of x , up to and including the term in x^2 .
Give each coefficient in its simplest form.

(5)

(b) Find the exact value of $(4 + 5x)^{\frac{1}{2}}$ when $x = \frac{1}{10}$

Give your answer in the form $k\sqrt{2}$, where k is a constant to be determined.

(1)

(c) Substitute $x = \frac{1}{10}$ into your binomial expansion from part (a) and hence find an approximate value for $\sqrt{2}$

Give your answer in the form $\frac{p}{q}$ where p and q are integers.

(2)



3.

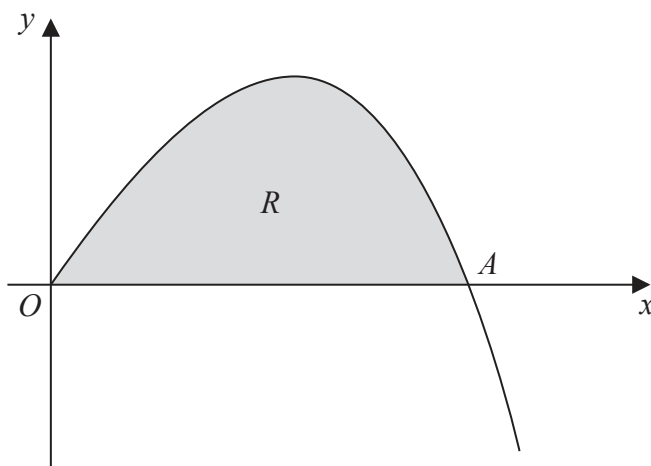


Figure 1

Figure 1 shows a sketch of part of the curve with equation $y = 4x - xe^{\frac{1}{2}x}$, $x \geq 0$

The curve meets the x -axis at the origin O and cuts the x -axis at the point A .

(a) Find, in terms of $\ln 2$, the x coordinate of the point A . (2)

(b) Find

$$\int xe^{\frac{1}{2}x} dx$$
(3)

The finite region R , shown shaded in Figure 1, is bounded by the x -axis and the curve with equation

$$y = 4x - xe^{\frac{1}{2}x}, \quad x \geq 0$$

(c) Find, by integration, the exact value for the area of R .
Give your answer in terms of $\ln 2$ (3)



4. With respect to a fixed origin O , the lines l_1 and l_2 are given by the equations

$$l_1: \mathbf{r} = \begin{pmatrix} 5 \\ -3 \\ p \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ 1 \\ -3 \end{pmatrix}, \quad l_2: \mathbf{r} = \begin{pmatrix} 8 \\ 5 \\ -2 \end{pmatrix} + \mu \begin{pmatrix} 3 \\ 4 \\ -5 \end{pmatrix}$$

where λ and μ are scalar parameters and p is a constant.

The lines l_1 and l_2 intersect at the point A .

(a) Find the coordinates of A . (2)

(b) Find the value of the constant p . (3)

(c) Find the acute angle between l_1 and l_2 , giving your answer in degrees to 2 decimal places. (3)

The point B lies on l_2 where $\mu = 1$

(d) Find the shortest distance from the point B to the line l_1 , giving your answer to 3 significant figures. (3)



Question 4 continued

Lined area for writing the answer to Question 4.



5. A curve C has parametric equations

$$x = 4t + 3, \quad y = 4t + 8 + \frac{5}{2t}, \quad t \neq 0$$

(a) Find the value of $\frac{dy}{dx}$ at the point on C where $t = 2$, giving your answer as a fraction in its simplest form.

(3)

(b) Show that the cartesian equation of the curve C can be written in the form

$$y = \frac{x^2 + ax + b}{x - 3}, \quad x \neq 3$$

where a and b are integers to be determined.

(3)



6.

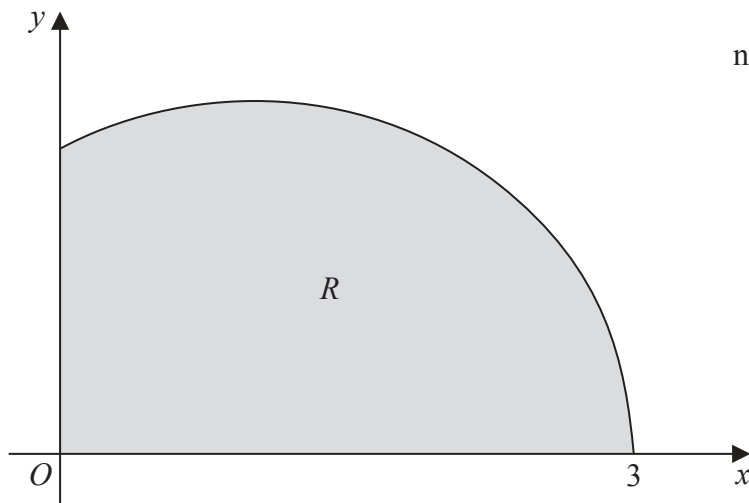
Diagram
not to scale

Figure 2

Figure 2 shows a sketch of the curve with equation $y = \sqrt{(3-x)(x+1)}$, $0 \leq x \leq 3$

The finite region R , shown shaded in Figure 2, is bounded by the curve, the x -axis, and the y -axis.

(a) Use the substitution $x = 1 + 2 \sin \theta$ to show that

$$\int_0^3 \sqrt{(3-x)(x+1)} \, dx = k \int_{-\frac{\pi}{6}}^{\frac{\pi}{2}} \cos^2 \theta \, d\theta$$

where k is a constant to be determined.

(5)

(b) Hence find, by integration, the exact area of R .

(3)



7. (a) Express $\frac{2}{P(P-2)}$ in partial fractions.

(3)

A team of biologists is studying a population of a particular species of animal.

The population is modelled by the differential equation

$$\frac{dP}{dt} = \frac{1}{2}P(P-2)\cos 2t, \quad t \geq 0$$

where P is the population in thousands, and t is the time measured in years since the start of the study.

Given that $P = 3$ when $t = 0$,

(b) solve this differential equation to show that

$$P = \frac{6}{3 - e^{\frac{1}{2}\sin 2t}}$$

(7)

(c) find the time taken for the population to reach 4000 for the first time.
Give your answer in years to 3 significant figures.

(3)



Question 7 continued

Handwritten answer area consisting of 30 horizontal lines.

Q7

(Total 13 marks)

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8.

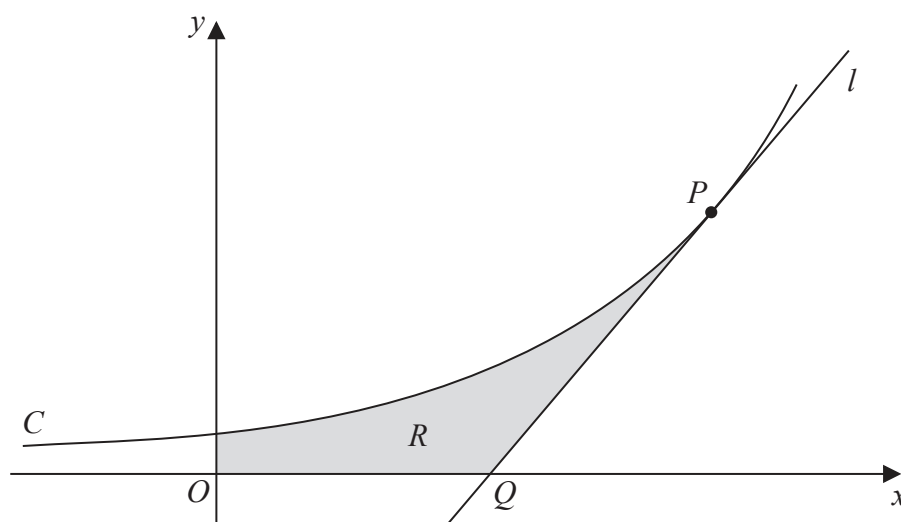


Diagram not to scale

Figure 3

Figure 3 shows a sketch of part of the curve C with equation

$$y = 3^x$$

The point P lies on C and has coordinates $(2, 9)$.

The line l is a tangent to C at P . The line l cuts the x -axis at the point Q .

- (a) Find the exact value of the x coordinate of Q . (4)

The finite region R , shown shaded in Figure 3, is bounded by the curve C , the x -axis, the y -axis and the line l . This region R is rotated through 360° about the x -axis.

- (b) Use integration to find the exact value of the volume of the solid generated.

Give your answer in the form $\frac{p}{q}$ where p and q are exact constants.

[You may assume the formula $V = \frac{1}{3}\pi r^2 h$ for the volume of a cone.] (6)



Question 8 continued

Lined writing area with 29 horizontal lines.

Q8

(Total 10 marks)

TOTAL FOR PAPER: 75 MARKS

END

